Example: With $f(x,y) = \frac{7}{8}(y^2+y+x)^4$

compute $f_{xyxyx}(2,0)$.

Solution 1: $f_x(x,y) = \frac{7}{8}4(y^2+y+x)^3 \cdot (1)$

$$f_{xy}(x,y) = \frac{7}{2}3(y^2+y+x)^2(2y+1)$$

$$f_{xyx}(x,y) = 21(y^2+y+x)(2y+1)$$

$$f_{xyxy}(x,y) = 21(2y+1)(2y+1) + 21(y^2+y+x)(2)$$

$f_{xyxyx}(x,y) = 42$. $f_{xyxyx}(2,0) = 42$.

Solution 2: $f_{xyxyx}(x,y) = f_{xxxyy}(x,y)$

$$f_{xx}(x,y) = \frac{21}{2}(y^2+y+x)^2$$

$$f_{xxx}(x,y) = 21(y^2+y+x)$$

$$f_{xxxy}(x,y) = 42y+21$$

$f_{xxxyy}(x,y) = 42$.

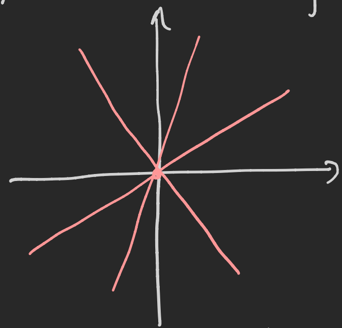
$f_{xxxyy}(2,0) = 42$.

(So we've verified Clairaut's thm in this specific example.)

When dealing with a limit problem...

Try some easy paths, e.g. for $\lim\limits_{(x,y)\to(0,0)}$

try $y=mx$:



Maybe also along $x=y^2$ or similar if e.g. denominator is x^4+y^8 etc..

If you ever find 2 different answers, you can conclude the limit <u>does not exist</u>.

However, no matter how many paths you try, even if they all give you the same answer, you <u>cannot</u> conclude the limit does exist.

Some methods for showing a limit <u>does</u> exist:

- switch to polar: especially if you see x^2+y^2 in denom.
- Squeeze Thm.
- "Relocating" to origin by change of vars:

$$\lim_{(x,y)\to(1,0)} \frac{xy-y}{(x-1)^2+y^2} = \lim_{(u,v)\to(0,0)} \frac{(u+1)v-v}{u^2+v^2}$$

$u=x-1$

$v=y$